CDI-II - prática, 9/3/21

A ⊂ $\mathbb{R}^n$ compacto se sse fechado e limitado

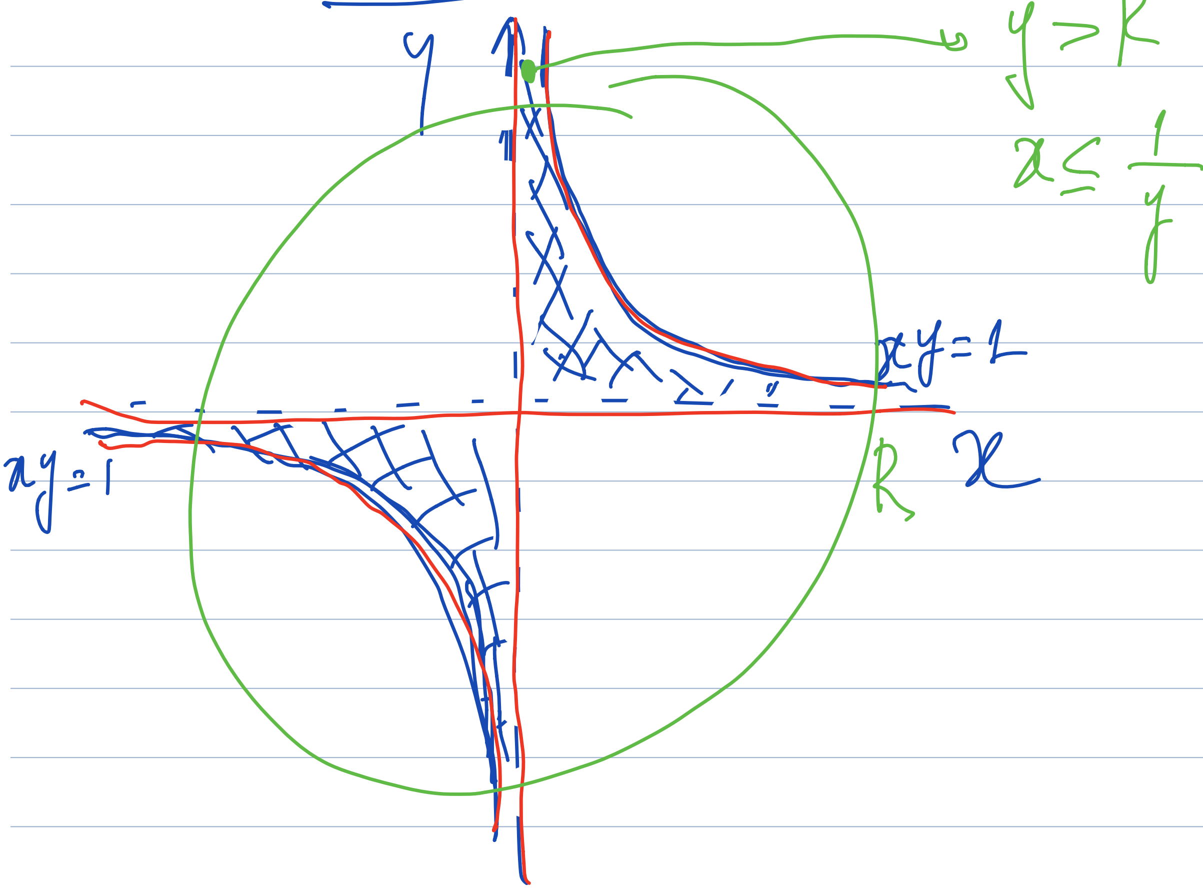
A ⊂ $\mathbb{R}^n$ limitado se

$\exists\, B_R(0) : A \subset B_R(0)$.

Ficha 2.

1-b) $\ln(xy) \le 0$

$$\boxed{0 < xy \le 1}$$



$$xy = 0 \iff x = 0 \lor y = 0$$

$$xy > 0 \iff \underline{(x > 0 \land y > 0)} \lor \underline{(x < 0 \land y < 0)}$$

$$2-d)\quad \left(\frac{x^2y}{x^2+y^2}\right)\left(\frac{\operatorname{sen}(x^2+y^2)}{x^2+y^2}\right)$$

2a)

Caso notável

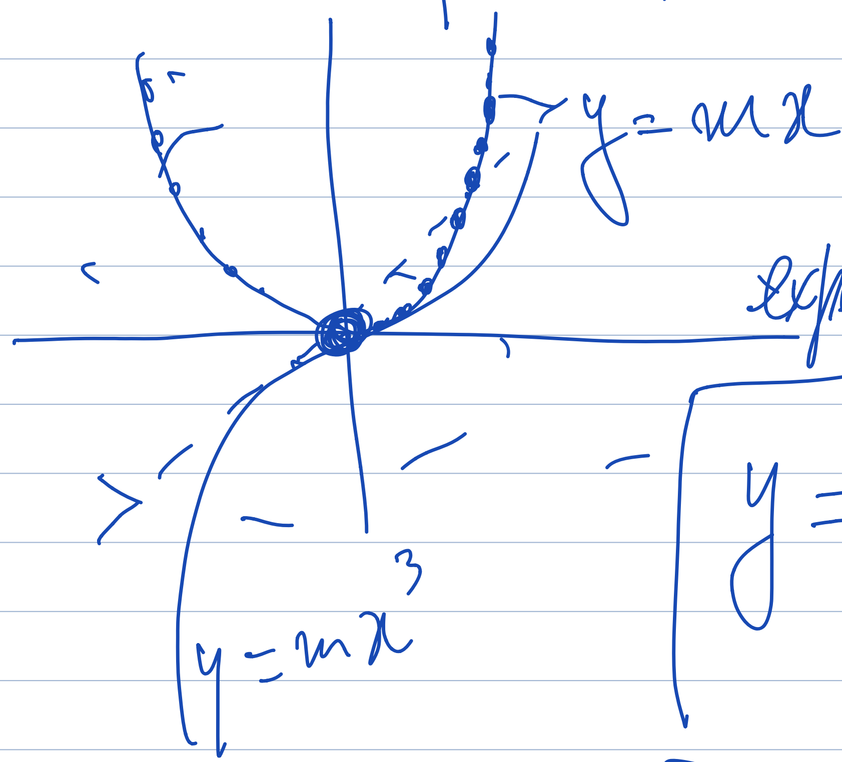
$R = x^2+y^2$

$$\frac{\operatorname{sen} R}{R}$$

$$\left|\frac{x^2y}{x^2+y^2}\right| = \underbrace{\frac{x^2}{x^2+y^2}}_{\leq 1}|y| \leq |y| \to 0$$

2-f) $x\ln(xy) = \underbrace{x\ln x} + \boxed{x\ln y}$?

notável!



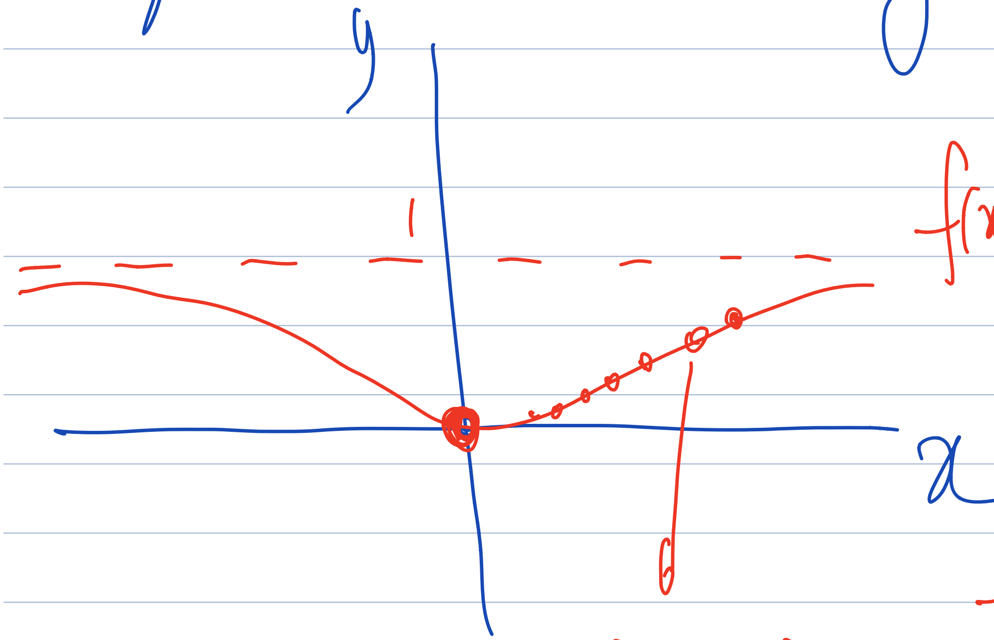
experimentar!

$$\boxed{y = mx^k \qquad k \neq 0}$$

$$x\ln(xy) = x\ln x + x\ln(mx^k)$$

$$= \underbrace{x\ln x} + \underbrace{x\ln m} + \underbrace{kx\ln x}$$

experimentar em $y = e^{-\frac{1}{x^2}}$!



$f(x) = e^{-\frac{1}{x^2}}$

$x \neq 0$

$\downarrow$

$f(0) = 0$

$(x, f(x)) \longrightarrow (0,0)$

$$x \ln(xy) = x\ln(x) + x \ln\left(e^{-\frac{1}{x^2}}\right)$$

$$= x \ln x + x\left(-\frac{1}{x^2}\right)$$

$$= x \ln x - \frac{1}{x} \uparrow \infty$$

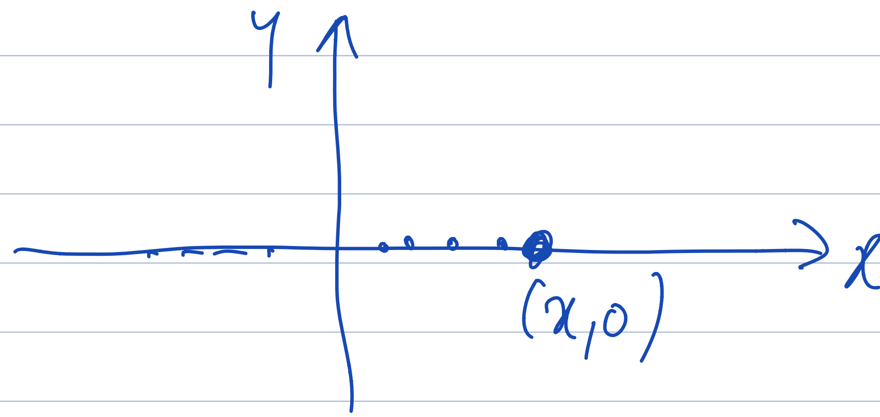
3-b) $\boxed{y=0} \Longrightarrow \dfrac{x}{\sqrt{x^2}} = \dfrac{x}{|x|}$

$$\frac{x}{|x|} = \begin{cases} 1 & \text{se } x>0 \\ -1 & \text{se } x<0 \end{cases}$$

3 candidatos a limite:

$0, \ -1, \ 1.$

$\therefore$ limite não existe. !



3-c) $\longleftarrow$ 2-e)

3-d) $$\underbrace{\frac{|x|}{\sqrt{x^2+y^2}}}_{\leq 1}|x| \leq |x| \to 0$$

$$|x| \leq \sqrt{x^2+y^2}$$

$\Longrightarrow$ f é contínua em $\mathbb{R}^2$

3-e) $$|\operatorname{sen}(\cdot)| \leq 1$$